A novel approach to determine optimum switching frequency of a conventional adsorption chiller

K.C.A. Alam, Y.T. Kang *, B.B. Saha 1, A. Akisawa, T. Kashiwagi

Department of Mechanical Systems Engineering, Tokyo University of A & T, 2-24-16 Naka-Cho, Koganei, Tokyo 184-8588, Japan

Received 16 March 2000

Abstract

This article investigates the effect of design parameters on the switching frequency of a conventional adsorption chiller with silica gel as adsorbent and water as adsorbate. It is well known that as the cycle time lengthens, the coefficient of performance (COP) rises but the cooling capacity lowers. Optimum cycle time is dependent on the requirements of COP and cooling capacity. A novel simulation technique that introduces a profit function is employed to determine the optimum switching frequency of an adsorption refrigeration system. The results show that optimum switching frequency is very sensitive to the heat exchanger’s design parameters. The design parameters are characterized by the number of transfer unit, NTU, the Biot number of adsorbent bed, Bi, the aspect ratio, AR, the ratio of the heat exchanger thickness to the radius of the fluid channel Hr, the fluid alpha number, \( \alpha_{f-a} \) and the inert material alpha number, \( \alpha_{m-a} \).

The optimum switching frequency increases with the increase of NTU, Hr and with the decrease of Bi, AR, \( \alpha_{m-a} \) and \( \alpha_{f-a} \).

© 2003 Elsevier Science Ltd. All rights reserved.

1. Introduction

In the promotion of environmentally friendly energy utilization systems, one major goal is to develop CFC-free refrigeration/heat pump systems that utilize waste heat or renewable energy sources. Adsorption cooling systems promise to provide a safe alternative to CFC-basis refrigeration devices. From this point of view, a number of researchers investigated the possibility of an adsorption heat-pumping/refrigeration system driven by waste heat or by renewable heat sources.
**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>heat transfer area</td>
</tr>
<tr>
<td>$AR$</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>$Bi$</td>
<td>Biot number</td>
</tr>
<tr>
<td>$c$</td>
<td>specific heat</td>
</tr>
<tr>
<td>$COP$</td>
<td>coefficient of performance</td>
</tr>
<tr>
<td>$D_a$</td>
<td>adsorbent thickness</td>
</tr>
<tr>
<td>$D_f$</td>
<td>fluid tube radius</td>
</tr>
<tr>
<td>$D$</td>
<td>$D_a + D_f$</td>
</tr>
<tr>
<td>$h_f$</td>
<td>heat transfer coefficient</td>
</tr>
<tr>
<td>$h_m$</td>
<td>mass transfer coefficient</td>
</tr>
<tr>
<td>$Hr$</td>
<td>heat exchanger thickness ratio</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>$KR$</td>
<td>thermal conductivity ratio</td>
</tr>
<tr>
<td>$L$</td>
<td>length of a bed</td>
</tr>
<tr>
<td>$L_v$</td>
<td>latent heat of refrigerant</td>
</tr>
<tr>
<td>$M$</td>
<td>mass</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>mass flow rate</td>
</tr>
<tr>
<td>NSCC</td>
<td>non-dimensional specific cooling capacity</td>
</tr>
<tr>
<td>NTU</td>
<td>number of transfer unit</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure</td>
</tr>
<tr>
<td>$p$</td>
<td>price ratio</td>
</tr>
<tr>
<td>Pf</td>
<td>non-dimensional profit function</td>
</tr>
<tr>
<td>$p1$</td>
<td>price of input heat</td>
</tr>
<tr>
<td>$p2$</td>
<td>price of heat output</td>
</tr>
<tr>
<td>$q$</td>
<td>concentration</td>
</tr>
<tr>
<td>$\bar{q}$</td>
<td>non-dimensional concentration</td>
</tr>
<tr>
<td>$q_e$</td>
<td>concentration equilibrium</td>
</tr>
<tr>
<td>$\bar{q_e}$</td>
<td>non-dimensional concentration equilibrium</td>
</tr>
<tr>
<td>$Q_{st}$</td>
<td>isosteric heat of adsorption</td>
</tr>
<tr>
<td>SCC</td>
<td>specific cooling capacity</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$t_{hc}$</td>
<td>switching time</td>
</tr>
<tr>
<td>$w$</td>
<td>non-dimensional switching frequency</td>
</tr>
<tr>
<td>$x, y$</td>
<td>coordinate axes</td>
</tr>
<tr>
<td>$X, Y$</td>
<td>non-dimensional coordinate axes</td>
</tr>
</tbody>
</table>

**Greek letters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{f,a}$</td>
<td>fluid alpha number</td>
</tr>
<tr>
<td>$\alpha_{r,a}$</td>
<td>refrigerant alpha number</td>
</tr>
</tbody>
</table>
Several kinds of heat pumping and refrigeration applications have been developed using various adsorbent and adsorbate pairs. Some typical achievements in adsorption heat pump/refrigeration systems [1–12] are presented in Table 1.

In adsorption heat pump/cooling systems, cycle time is one of the main operating conditions. It is well known that an excessively short cycle time is responsible for lower cooling capacity and coefficient of performance (COP), and a longer cycle time may cause a lower performance. An experimental analysis of Boelman et al. [8] shows that there exists an optimum cycle time for a maximum cooling capacity, but COP increases monotonically as cycle time increases, at least until 1300 s. Saha et al. [7] and recently, Chua et al. [13] have investigated the effect of cycle time on the system performance by proposing a simulation model that agrees with the results of Boelman et al. [9]. These three analyses were all based on the same machine, installed at Tokyo University of Agriculture and Technology. They did not consider the different configurations of the heat exchangers. A numerical study by Haji and Worek [14] shows that an excessively long cycle time also is responsible for a lower COP. The effect of switching speed on the system performance was also studied by Zheng et al. [15,16]. They showed that there is an optimum switching speed to optimize COP and cooling capacity, which are different, for a given design and operating conditions. Later, Amar et al. [1] proved that for the longer cycle times, the high-
Table 1
Developments in adsorption heat pump/refrigeration systems

<table>
<thead>
<tr>
<th>Adsorbent/adsorbate</th>
<th>System type</th>
<th>Source</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activated carbon/ammonia</td>
<td>Regenerative system</td>
<td>Amar et al. [1], Fuller et al. [2]</td>
<td>Two bed system</td>
</tr>
<tr>
<td>Activated carbon/ammonia</td>
<td>Regenerative system</td>
<td>Jones [3]</td>
<td>Four bed system</td>
</tr>
<tr>
<td>Activated carbon/methanol</td>
<td>Regenerative system</td>
<td>Pons and Guilleminot [4]</td>
<td>Solar driven ice maker</td>
</tr>
<tr>
<td>Calcium chloride/methanol</td>
<td>Intermittent adsorption</td>
<td>Lai et al. [5]</td>
<td>Chemical heat pump</td>
</tr>
<tr>
<td>Complex compounds/salts</td>
<td>Intermittent adsorption</td>
<td>Beijer and Horsman [6]</td>
<td>Promising uses: vehicles and residential air conditioning</td>
</tr>
<tr>
<td>Silica gel/water</td>
<td>Intermittent adsorption</td>
<td>Saha et al. [7], Boelman et al. [8]</td>
<td>Waste heat driven cycle</td>
</tr>
<tr>
<td>Silica gel/water</td>
<td>Three stage cycle</td>
<td>Saha et al. [9]</td>
<td>Waste heat driven cycle; ( T_{\text{regeneration}} ) is very low</td>
</tr>
<tr>
<td>Zeolite/ammonia</td>
<td>Intermittent system</td>
<td>Critoph and Turner [10]</td>
<td>( T_{\text{regeneration}} ) is very high</td>
</tr>
<tr>
<td>Zeolite composites/water</td>
<td>Intermittent adsorption</td>
<td>Guilleminot et al. [12]</td>
<td>Composites: (a) 65% zeolite + 35% metallic foam and (b) 70% zeolite + 30% natural expanded graphite</td>
</tr>
</tbody>
</table>

temperature front breaks through the bed and the heat is transferred directly from the boiler to the cooler, causing the COP to decrease. Recently, Alam et al. [17] investigated the influence of heat exchanger design parameters on the system performance as well as on the switching frequency (cycle time). They showed that the optimum switching frequency is strongly dependent on the heat exchanger design parameters. They also showed that the COP and cooling capacity are not optimized at a single switching frequency.

An adsorption heat pump/cooling system such as silica gel–water adsorption system is applied to recover low grade waste heat or to utilize the renewable energy sources such as solar heat. In this type of system, manufacturers always think that heat is available and do not need to pay for it. This is why they always prefer the cycle time where the cooling/heating capacity is maximized. But, practical situations reveal that heat is not always free and available. At least, there is some maintenance cost to utilize the heat. Therefore, scientists and engineers should consider the overall performance of the system. From this context, a two bed adsorption refrigeration system has been considered based on the model of Alam et al.[17] and a new profit function as a performance index is introduced to determine a single optimum point for switching frequency in the present study. A parametric analysis has been conducted to obtain the optimum switching frequency for different design and operating conditions.
2. Mathematical modeling

In adsorption heat pump/refrigeration systems, the vapor-compression cycle is carried out by adsorbent bed heat exchangers. The schematic diagram of the different positions of an adsorbent bed heat exchanger has been presented in Fig. 1. In a two bed adsorption cooling system, two adsorbent bed heat exchangers are used alternatively for the adsorption and regeneration process in a cyclic manner. Adsorbent beds are filled out with the adsorbent materials, silica gel. The system is driven by heating one bed and simultaneously cooling the second bed by the heat transfer fluid, water. The cycle description of the two bed adsorption refrigeration/heat pump is available elsewhere in Refs. [14–17].

Most of the numerical analyses [1–2,4,14–17] on the adsorption cooling system had been conducted by considering a one-dimensional (axial direction) heat equation for the heat transfer fluid path. With the one-dimensional analysis, one cannot analyze the effect of aspect ratio and heat exchanger thickness ratio. One has to use two-dimensional analysis if one wants to analyze the effect of those parameters. The fluid side is a vital part of the adsorbent bed heat exchangers.

![Fig. 1. (a) Schematic of an adsorbent bed heat exchanger; (b) schematic of side view of the adsorbent bed heat exchanger; (c) schematic of a flow channel.](image-url)
Therefore, it is desirable to investigate the design parameters of the fluid side in more detail. However, very little literature has been found on the parametric study of the fluid side. From this context, in the present investigation, two-dimensional heat equations are considered for both the fluid and adsorbent sides. The following assumptions are made for mathematical modeling of the two bed adsorption systems: (i) the bed has sufficient unoccupied space, and vapor pressure throughout the bed is uniform but varies with time, (ii) the particles are small enough to be regarded as saturated, (iii) constant thermophysical properties, (iv) the refrigerant vapor behaves as an ideal gas and (v) the adsorbed phase behaves as a liquid. According to these assumptions, the energy equation for the heat transfer fluid can be written as

\[ M_f c_f \frac{\partial T_f}{\partial t} = -\dot{m}_f c_f L \frac{\partial T_f}{\partial x} + k_f A_f \left( \frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) \]  

The energy equation for the bed is expressed as

\[ M_a c_a \left( 1 + \frac{c_r q}{c_a} + \frac{M_m c_m}{M_a c_a} \right) \frac{\partial T_b}{\partial t} - \rho_a A_a L Q_w \frac{\partial q}{\partial t} = k_{eff} A_f \left( \frac{\partial^2 T_b}{\partial x^2} + \frac{\partial^2 T_b}{\partial y^2} \right) \]

In this analysis, it is assumed that the rate of change of moisture content in the bed is proportional to the difference between the equilibrium and the actual moisture content, and is expressed as follows using the mass transfer coefficient \( h_m \):

\[ M_a \frac{\partial q}{\partial t} = h_m (q_e - q), \]

where \( q_e \) is the equilibrium uptake at \( (T_s, T_e) \) and is given in Refs. [18] and [19]. The initial and boundary conditions for the present problems are as follows.

For the regeneration process,

\[ T_f(x, y, 0) = T_b(x, y, 0) = T_c, \]

\[ T_f - T_b = \frac{\partial T_b}{\partial x} = 0 \text{ at } x = 0, \quad \frac{\partial T_f}{\partial x} = \frac{\partial T_b}{\partial x} = 0 \text{ at } x = L, \]

\[ \frac{\partial T_f}{\partial y} = 0 \text{ at } y = 0, \quad \frac{\partial T_f}{\partial y} = h_f (T_b - T_f) \text{ at } y = D_f, \]

\[ k_{eff} \frac{\partial T_b}{\partial y} = h_l (T_b - T_f) \text{ at } y = D_f, \quad \frac{\partial T_b}{\partial y} = 0 \text{ at } y = D = D_f + D_a \]

and

\[ q = q_e = q_e(T_c, T_{eva}) = q_{max} \text{ at } t = 0 \]

For the adsorption process,

\[ T_f(x, y, t_{hc}) = T_b(x, y, t_{hc}) = T_h, \]

\[ T_f - T_c = \frac{\partial T_b}{\partial x} = 0 \text{ at } x = L, \quad \frac{\partial T_f}{\partial x} = \frac{\partial T_b}{\partial x} = 0 \text{ at } x = 0, \]
\begin{equation}
\frac{\partial T_f}{\partial y} = 0 \quad \text{at } y = 0, \quad k_f \frac{\partial T_f}{\partial y} = h_f (T_b - T_f) \quad \text{at } y = D_f,
\end{equation}
\begin{equation}
k_{\text{eff}} \frac{\partial T_b}{\partial y} = h_f (T_b - T_f) \quad \text{at } y = D_f, \quad \frac{\partial T_b}{\partial y} = 0 \quad \text{at } y = D = D_f + D_a
\end{equation}

and \( q \) is known from the previous period.

The following groups of transformation are introduced into Eqs. (1)–(12) to normalize the governing equations,

\begin{equation}
\theta = \frac{T-T_c}{T_h-T_c}, \quad \tau = \frac{t}{t_{\text{tc}}}, \quad X = \frac{x}{L}, \quad Y = \frac{y}{D} \quad \text{and} \quad \bar{q} = \frac{q}{q_{\text{max}} - q_{\text{min}}}
\end{equation}

Therefore, the non-dimensional energy equations for the heat transfer fluid and adsorbent bed are as follows:

\begin{equation}
\alpha_r \frac{\partial \theta_f}{\partial \tau} = -\frac{\partial \theta_f}{\partial X} + \frac{\text{NTU} \cdot \text{KR}}{\text{Bi}(1 + \text{Hr}) \text{AR}} \left( \frac{\partial^2 \theta_f}{\partial X^2} + \text{AR} \frac{\partial^2 \theta_f}{\partial Y^2} \right)
\end{equation}

\begin{equation}
\left[ (1 + \alpha_{r.a} \theta + \alpha_{r.a} \frac{\partial q}{\partial \tau}) \frac{\partial \theta_b}{\partial \tau} - \frac{\partial q}{\partial \tau} \right] \frac{\text{w} \cdot \text{Bi}(1 + \text{Hr}) \cdot \text{Hr} \cdot \text{AR}}{\text{NTU}} = \frac{\partial^2 \theta_b}{\partial X^2} + \text{AR}^2 \frac{\partial^2 \theta_b}{\partial Y^2}
\end{equation}

where \( \theta_f \) and \( \theta_b \) are the non-dimensional temperature of the fluid and adsorbent bed, respectively, at non-dimensional time, \( \tau \). Other parameters are defined in Table 2 and in the nomenclature.

The refrigerant mass balance can be expressed as

\begin{equation}
\frac{d \bar{q}}{d \tau} = \mu (\bar{q}_e - \bar{q})
\end{equation}

where \( \bar{q} \) is the non-dimensional refrigerant concentration at the non-dimensional time \( \tau \), \( \bar{q}_e ( = q_e / q_{\text{max}} ) \) is the non-dimensional equilibrium state.

Initial and boundary conditions are as follows.

For the regeneration process,

\begin{equation}
\theta_f(X,Y,0) = \theta_b(X,Y,0) = 0
\end{equation}

\begin{equation}
\theta_f - 1 = \frac{\partial \theta_b}{\partial X} = 0 \quad \text{at } X = 0, \quad \frac{\partial \theta_f}{\partial X} = \frac{\partial \theta_b}{\partial X} = 0 \quad \text{at } X = 1
\end{equation}

\begin{equation}
\frac{\partial \theta_f}{\partial Y} = 0 \quad \text{at } Y = 0, \quad \frac{\partial \theta_f}{\partial Y} = \frac{\text{Bi}(1 + \text{Hr}) (\theta_b - \theta_f)}{\text{KR}} \quad \text{at } Y = \frac{\text{Hr}}{1 + \text{Hr}}
\end{equation}

\begin{equation}
\frac{\partial \theta_b}{\partial Y} = 0 \quad \text{at } Y = 1, \quad \frac{\partial \theta_b}{\partial Y} = \text{Bi}(1 + \text{Hr}) (\theta_b - \theta_f) \quad \text{at } Y = \frac{\text{Hr}}{1 + \text{Hr}}
\end{equation}

and

\begin{equation}
\bar{q} = 1
\end{equation}

For the adsorption process,


Table 2
List of non-dimensional parameters used in the analysis

<table>
<thead>
<tr>
<th>Non-dimensional parameter</th>
<th>Mathematical expression</th>
<th>Definition</th>
<th>Source author</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching frequency</td>
<td>( w = M_a c_a / \dot{m}<em>f c_f \tau_h = t_h / t</em>{hc} )</td>
<td>Ratio of the required time to take the heat capacitance of adsorbent material by the heat transfer fluid to the switching time, ( t_{hc} )</td>
<td>Zheng et al. [15,16]</td>
</tr>
<tr>
<td>Number of transfer unit</td>
<td>( \text{NTU} = h_t A_t / \dot{m}_f c_f )</td>
<td>Ratio of the heat transfer at the interface of the fluid/tube to the advection of energy in the fluid</td>
<td>Zheng et al. [15,16]</td>
</tr>
<tr>
<td>Bed Biot number</td>
<td>( \text{Bi} = h_t D_a / k_{eff} )</td>
<td>Ratio of conductive resistance in the adsorbent layer to the convective resistance in the heat transfer fluid</td>
<td>Zheng et al. [15,16]</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>( \text{AR} = L / D )</td>
<td>Ratio of the length of the heat exchanger to half of the width of the heat exchanger</td>
<td>Ellis and Wepfer [20]</td>
</tr>
<tr>
<td>Heat exchanger thickness ratio</td>
<td>( \text{Hr} = D_f / D_a )</td>
<td>Ratio of the radius of the heat transfer fluid channel to the thickness of adsorbent materials</td>
<td>Alam et al. [17]</td>
</tr>
<tr>
<td>Inert material alpha number</td>
<td>( \alpha_{m-a} = M_m c_m / M_a c_a )</td>
<td>Heat capacitance ratio of the inert mass to the adsorbent mass</td>
<td>Zheng et al. [15,16]</td>
</tr>
<tr>
<td>Fluid alpha number</td>
<td>( \alpha_{f-a} = M_f c_f / M_a c_a )</td>
<td>Heat capacitance ratio of the resident heat transfer fluid mass to the adsorbent mass</td>
<td>Zheng et al. [15,16]</td>
</tr>
<tr>
<td>Refrigerant alpha number</td>
<td>( \alpha_{r-a} = (c_r / c_a) q_{max} )</td>
<td>Specific heat capacitance ratio of the refrigerant to the adsorbent material</td>
<td>Zheng et al. [15,16]</td>
</tr>
<tr>
<td>Thermal conductivity ratio</td>
<td>( \text{KR} = k_f / k_{eff} )</td>
<td>Ratio of the thermal conductivity of the resident heat transfer fluid to the effective thermal conductivity of adsorbent bed</td>
<td>Shelton et al. [21]</td>
</tr>
<tr>
<td>Non-dimensional mass transfer coefficient</td>
<td>( \mu = h_t f_{hc} / M_a )</td>
<td>Mass ratio of the total refrigerant adsorbed by the adsorbent to the dry adsorbent</td>
<td>Alam et al. [17]</td>
</tr>
<tr>
<td>Adsorbent beta number</td>
<td>( \beta = Q_{st} q_{max} / c_a (T_h - T_c) )</td>
<td>Heat ratio of the adsorption process to the required heat of adsorbent material to change its temperature from ( T_c ) to ( T_h )</td>
<td>Zheng et al. [15,16]</td>
</tr>
<tr>
<td>Lambda number</td>
<td>( \lambda = L_s / Q_{st} )</td>
<td>Ratio of the latent heat evaporation to adsorption heat</td>
<td>Zheng et al. [15,16]</td>
</tr>
</tbody>
</table>

\[
\theta_i(X, \ Y, \ 1) = \theta_b(X, \ Y, \ 1) = 1
\]

\[
\theta_f = \frac{\partial \theta_b}{\partial X} = 0 \quad \text{at} \quad X = 1, \quad \frac{\partial \theta_f}{\partial X} = \frac{\partial \theta_b}{\partial X} = 0 \quad \text{at} \quad X = 0
\]

\[
\frac{\partial \theta_f}{\partial Y} = 0 \quad \text{at} \quad Y = 0, \quad \frac{\partial \theta_f}{\partial Y} = \frac{\text{Bi}(1 + \text{Hr})}{\text{KR}}(\theta_b - \theta_f) \quad \text{at} \quad Y = \frac{\text{Hr}}{1 + \text{Hr}}
\]

\[
\frac{\partial \theta_b}{\partial Y} = 0 \quad \text{at} \quad Y = 1, \quad \frac{\partial \theta_b}{\partial Y} = \frac{\text{Bi}(1 + \text{Hr})(\theta_b - \theta_f)}{1 + \text{Hr}} \quad \text{at} \quad Y = \frac{\text{Hr}}{1 + \text{Hr}}
\]

where the following non-dimensional variables are used in the present analysis:
3. System performance equations

The system performance of an adsorption cooling unit is measured mainly by the following two parameters, (i) COP, the coefficient of performance and (ii) SCC (kg/kg of the dry adsorbent), specific cooling capacity. The COP and SCC can be estimated by the following equations:

\[
\text{COP} = \frac{Q_{\text{eva}}}{Q_{\text{in}}} \quad (27)
\]

and

\[
\text{SCC} = \frac{Q_{\text{eva}}}{M_{a} \dot{m}_{hc}} \quad (28)
\]

The energy input during a half cycle can be measured as

\[
Q_{\text{in}} = \dot{m}_{hc} \int_{0}^{t_{hc}} (T_{\text{in}} - T_{\text{out}}) dt, \quad (29)
\]

and the heat extracted from the evaporator is calculated as

\[
Q_{\text{eva}} = \int_{0}^{t_{hc}} [L_{v}(T_{\text{eva}}) - c_{r}(T_{\text{con}} - T_{\text{eva}})] M_{a} \frac{dq}{dt} dt. \quad (30)
\]

In terms of non-dimensional form, Eqs. (29) and (30) can be rewritten as

\[
\bar{Q}_{\text{in}} = \frac{Q_{\text{in}}}{\dot{m}_{hc}(T_{H} - T_{C})} = \int_{0}^{1} (1 - \theta_{\text{out}}) d\tau, \quad (31)
\]

and

\[
\bar{Q}_{\text{eva}} = \frac{Q_{\text{eva}}}{\dot{m}_{hc}(T_{H} - T_{C})} = \frac{w[\lambda \beta - \alpha_{r,a}(\theta_{\text{con}} - \theta_{\text{eva}})](\bar{q}|_{\tau=0} - \bar{q}|_{\tau=1})}{\int_{0}^{1} (1 - \theta_{\text{out}}) d\tau}. \quad (32)
\]

Therefore, COP can be written as

\[
\text{COP} = \frac{\bar{Q}_{\text{eva}}}{\bar{Q}_{\text{in}}} = \frac{w[\lambda \beta - \alpha_{r,a}(\theta_{\text{con}} - \theta_{\text{eva}})](\bar{q}|_{\tau=0} - \bar{q}|_{\tau=1})}{\int_{0}^{1} (1 - \theta_{\text{out}}) d\tau} \quad (33)
\]

According to Zheng et al. [15], the non-dimensional specific cooling capacity NSCC is as follows:

\[
\text{NSCC} = \frac{\text{SCC}}{\dot{m}_{hc}(T_{H} - T_{C})/M_{a}} \quad (34)
\]
or
\[ \text{NSCC} = w(\lambda \beta - \alpha_{\text{con}} - \theta_{\text{eva}})(q|_{c=0} - q|_{c=1}) \] (35)

4. Profit function

Generally, the performance of a heat pump/refrigeration system is characterized by the two performance parameters, namely, COP and SCC. The higher the SCC, the better the capacity of the system, while COP measures the efficiency of the system. Therefore, higher values of both COP and SCC are required in real systems. If the input heat is available and free, then the system can be optimized with an optimum SCC. However, if the heat is not free and available, a new performance equation is needed. An optimum condition of a system can be determined by considering the requirements of COP and SCC. The requirements of COP and SCC depend on the heat input/output price and the availability of heat input. The new performance equation is introduced as the difference between the output price of heat and input price of heat, and this performance index can be denoted as the profit function. Therefore, the profit function can be expressed mathematically as

\[ \text{Profit} = \bar{Q}_{\text{eva}}p_1 - \bar{Q}_{\text{in}}p_2 \] (36)

where \( p_1 \) and \( p_2 \) are the unit output and input price of non-dimensional heat, respectively. In terms of non-dimensional form, the profit function can be written as

\[ Pf = \bar{Q}_{\text{eva}} - \bar{Q}_{\text{in}}p \] (37)

where \( Pf = \text{Profit}/p_1 \) and \( p = p_2/p_1 \) is the price ratio. A lower \( p \) gives a higher Pf. Under the waste heat recovery condition, the value of \( p \) will be zero.

5. Solution methodologies

The non-dimensional governing equations have been solved numerically by employing the alternating direction implicit (ADI) method. The entire computational domain is divided into a number of equal step discrete elements. The quadratic upstream differencing scheme (QUDS) [22] has been applied to approximate the spatial first derivative term while a second order central difference scheme has been employed to estimate the the second order spatial derivative term.

The four thermodynamic steps have been taken into account in the solution procedure. The solution techniques applied in this analysis are divided mainly into two strategies; one is the pressurization/depressurization process and the other is the constant pressure process. During the pressurization/depressurization process, the mass transfer into the system is assumed to be constant, i.e. no vapor mass is allowed to enter/leave the system. The mass balance has been checked to calculate the bed pressure. The convergence criterion for all cases used in this program is \( 10^{-6} \). The values taken for the base run of this analysis are presented in Table 3.
Table 3
Base run parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$</td>
<td>30 °C</td>
</tr>
<tr>
<td>$T_h$</td>
<td>80 °C</td>
</tr>
<tr>
<td>$T_{con}$</td>
<td>30 °C</td>
</tr>
<tr>
<td>$T_{eva}$</td>
<td>14 °C</td>
</tr>
<tr>
<td>NTU</td>
<td>50</td>
</tr>
<tr>
<td>$Kr$</td>
<td>2.0</td>
</tr>
<tr>
<td>$Bi$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.34</td>
</tr>
<tr>
<td>AR</td>
<td>10</td>
</tr>
<tr>
<td>$\alpha_{m-a}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$H_r$</td>
<td>0.5</td>
</tr>
<tr>
<td>$q_{max}$</td>
<td>0.34 kg/kg</td>
</tr>
<tr>
<td>$Q_{st}$</td>
<td>2800 kJ/kg</td>
</tr>
<tr>
<td>$L_e$</td>
<td>2500 kJ/kg</td>
</tr>
<tr>
<td>$q_{max}$</td>
<td>0.34 kg/kg</td>
</tr>
</tbody>
</table>

6. Results and discussions

In the present article, a profit function has been imposed to investigate the optimum switching frequency. A set of non-dimensional design parameters is defined and the effect of those parameters on the profit function as well as on the optimum switching frequency has been analyzed.

The COP and NSCC for the base run parameters have been presented in Fig. 2. From this figure, it can be seen that the COP and NSCC are not optimized at the same point. The optimum switching frequency for COP ($w_{\text{COP,opt}}$) is lower than that for NSCC ($w_{\text{NSCC,opt}}$). In the words, a longer cycle time is needed to achieve the maximum COP compared to that needed to achieve the maximum NSCC. Fig. 2 also reveals that an excessively long cycle time may cause deterioration of system performance. Therefore, a profit function is adopted to determine the single optimum value of switching frequency ($w_{\text{opt}}$). The effect of different design parameters on the profit function as well as on $w_{\text{opt}}$ is discussed in the following paragraphs.

The effect of price ratio, $p$, on the non-dimensional profit function, $P_f$, is illustrated in Fig. 3. It is observed that an increase in the price ratio leads to a decrease in $P_f$. It is also observed that the optimum switching frequency, $w_{\text{opt}}$, increases as $p$ decreases. It may also be seen that for the waste heat recovery case ($p = 0.0$), $w_{\text{opt}} = w_{\text{NSCC,opt}}$. This is in accordance with the real situation. If there is no cost of input variables in a system, the maximum output is always preferable. That is why the manufacturers of adsorption chillers (stated in the literature of Chua et al. [13]) choose the cycle time as the time where cooling capacity is maximized. It can also be seen that the value of $w_{\text{opt}}$ is always between $w_{\text{COP,opt}}$ and $w_{\text{NSCC,opt}}$. The higher the price ratio, the lower the $w_{\text{opt}}$ and $P_f$. In other words, $w_{\text{opt}}$ approaches $w_{\text{COP,opt}}$ if $p$ tends to infinity. It is found that if $p$ is greater than COP, $P_f$ will take a negative value. Therefore, it may be concluded that the price ratio, $p$,
is always less than COP. In this analysis, the value of \( p \) is set at 0.2 to show the effect of heat exchanger design parameters on the optimum switching frequency, \( w_{\text{opt}} \).

The number of transfer unit, NTU, is one of the most important design parameters of any type of heat exchanger. The value of NTU directly affects system performance. Fig. 4 depicts the effect of NTU on the profit function as well as on the optimum switching frequency, \( w_{\text{opt}} \). From this figure, it can be seen that Pf increases with the increase of NTU. Usually, an increase in NTU leads to an increase in both COP and NSCC, which causes Pf to increase. Fig. 4 also shows that the rate of increase of Pf decreases as NTU increases. Therefore, it may be concluded that there should be an optimum value of NTU. In the present case, the optimum value of NTU is estimated as 50 for the variation limit of Pf less than 5%. In Fig. 4, \( w_{\text{opt}} \) lies between \( w_{\text{COP,opt}} \) and \( w_{\text{NSCC,opt}} \) and \( w_{\text{opt}} \) increases with the increase of NTU.

Another important design parameter of an adsorbent heat exchanger is Biot number, Bi. It represents the ratio of convection heat transfer to the conduction heat transfer in the heat
exchanger. The influence of Bi on Pf as well as on the $w_{\text{opt}}$ is presented in Fig. 5. One can see that an increase in Bi causes Pf to decrease. The effect of Bi on heat input and output as well as on the COP and NSCC is very complicated. The effect of Bi on both COP and NSCC was explained elaborately by Zheng et al. [15] and Alam et al. [17]. They showed that both COP and NSCC decrease with the increase of Bi. That means, a higher value of Bi gives a lower heat output, which causes Pf to decrease. The optimum switching frequency, $w_{\text{opt}}$, also decreases as Bi increases. In Fig. 5, one may observe that the rate of change of improvement in profit is very small, when Bi changes its value from 0.5 to 0.25. Therefore, it may be concluded that there should be an optimum value of Bi for given thermal and geometric conditions. For the present base run condition, the optimum value of Bi is approximated as 0.25 for the variation limit of Pf less than 5%.

The heat exchanger’s thickness ratio, Hr, is defined here as the ratio of the radius of the fluid channel to adsorbent bed thickness. The effect of Hr on the profit function, Pf, is illustrated in Fig. 6. It is seen that an increase in Hr leads to increase in both Pf and $w_{\text{opt}}$. An increase in Hr is analogous to a decrease in the adsorbent bed thickness or to an increase in the radius of heat transfer fluid channel. Therefore, it can be concluded that the thinner the adsorbent bed, the higher
the profit, which is a reasonable result, as we observe the results for the different Bi values in Fig. 5. Reduction of adsorbent bed thickness without changing the adsorbent amount causes the conductive resistance in the adsorbent bed to increase. A higher value of conductive resistance in adsorbent reactor results in lower heat and mass transfer performance within the adsorbent material, which is responsible for lower refrigeration effect and less profit. An increase in \( w_{\text{opt}} \) with the increase of \( H_r \) is observed in the same figure. The optimum value of \( H_r \) for the present base run condition is estimated as 0.5.

Fig. 7 shows the effect of aspect ratio, \( \alpha \), on the profit function as well as on the optimum switching frequency, \( w_{\text{opt}} \). From this figure, it can be seen that \( Pf \) is inversely proportional to the aspect ratio. In other words, \( Pf \) decreases with the increase of \( \alpha \). An increase in \( \alpha \) is equivalent to an increase in the length of the bed relative to the bed width. An increase of bed length with a fixed amount of adsorbent material needs more heat input to adsorb/desorb the same amount of refrigerant, which in turn causes performance to decrease, and as a result, lower profit. An increase in \( w_{\text{opt}} \) with the decrease of \( \alpha \) is also observed in the same figure. It can be seen that when changing \( \alpha \) from 10 to 5, \( Pf \) improves by 23%. But \( Pf \) improves by only 4% if \( \alpha \) changes its value from 10 to 5. Therefore, the optimum value of \( \alpha \) for the present analysis is considered as 5.0.

The influence of inert material alpha number, \( \alpha_{m-a} \), on \( Pf \) is seen in Fig. 8. It is seen that profit increases as \( \alpha_{m-a} \) decreases. The increase of \( \alpha_{m-a} \) is equivalent to an increase in the mass of inert material in adsorber relative to the adsorbent mass. A higher value of \( \alpha_{m-a} \) causes energy to be used to cool or heat the structure, which leads to deterioration of system performance. And this is responsible for less profit. It is also seen that a lower \( \alpha_{m-a} \) gives a higher \( w_{\text{opt}} \). In Fig. 8, it may be seen that the improvement rate of \( Pf \) is less than 5% if \( \alpha_{m-a} \) is less than 0.1. Therefore, the optimum value of \( \alpha_{m-a} \) is estimated as 0.1 for the present analysis.

The influence of fluid alpha number, \( \alpha_{f-a} \), on both \( Pf \) and \( w_{\text{opt}} \) is presented in Fig. 9. One may see that the profit improves slightly with the decrease of \( \alpha_{f-a} \). An increase in \( \alpha_{f-a} \) is analogous to an increase in the amount of heat transfer fluid in the flow channel of the adsorbent bed heat exchangers. If a large amount of heat transfer fluid is used to heat up or cool down a fixed amount of adsorbent material, it may cause waste of energy, resulting in the decrease of system perform-
Figs 8 and 9. Pf versus switching frequency, $w$ for different $\alpha_{m-a}$ and $\alpha_{f-a}$.

Fig. 8. Pf versus switching frequency, $w$ for different $\alpha_{m-a}$.

Fig. 9. Pf versus switching frequency, $w$ for different $\alpha_{f-a}$.

...ance as well as profit. Usually, $w_{opt}$ increases as Pf increases. The same effect is observed for $w_{opt}$ in Fig. 9. $w_{opt}$ increases with the decrease of $\alpha_{f-a}$ just as Pf increases with the decrease of $\alpha_{f-a}$. Since the variation effect of $\alpha_{f-a}$ on Pf is negligible, the optimum value of $\alpha_{f-a}$ is chosen as 0.1 for the present base run condition.

7. Conclusions

A simulation model has been derived for the conventional two bed adsorption refrigeration system and a profit function is introduced considering the price of heat input and output to determine the optimum switching frequency. Based on the parametric study, the following conclusions can be made:

(i) COP, NSCC and profit are very sensitive to the switching frequency. COP, NSCC and profit may deteriorate seriously if the system is not operated at optimum switching frequency.
(ii) Profit function, Pf, as well as optimum switching frequency, \( w_{\text{opt}} \), increases with the decrease of price ratio, \( p \). The system will not be profitable if the price ratio is greater than COP.

(iii) There should be an optimum switching frequency, \( w_{\text{opt}} \), for a fixed price ratio, \( p \), and for a given set of design parameters of an adsorption cooling system. \( w_{\text{opt}} \) always lies between \( w_{\text{COP,opt}} \) and \( w_{\text{NSCC,opt}} \).

(iv) Profit as well as optimum switching frequency, \( w_{\text{opt}} \), increases as NTU, Hr increase and decreases with the increase of Bi, AR, \( \alpha_{\text{m-a}} \) and \( \alpha_{\text{f-a}} \).

(v) We have successfully quantified the effect of design parameters on the switching frequency of a two bed adsorption chiller with silica gel–water.

References


